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First-Order Models for Satellite Survivability Optimization

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Methods for optimizing military satellite constellation sizing for primary mission performance need to be integrated with the sizing of satellite self-protection equipment. First-order models are developed for this purpose that consider all-out assault and attrition-type attacks. Optimization criteria such as minimizing constellation mass and maximizing the mean number of engagements required to kill a satellite are employed.

Introduction

As military applications in space increase, there is an ongoing need for satellite self-protection. The more traditional threat has been based on the assumption that an all-out assault would be employed to poke a "hole" in the portion of a constellation above a given geographic area, to clear the way for a large-scale attack. With current developments in the world order, this scenario is less likely than before. The emerging but limited capabilities of third-world countries now require that attrition-type attacks be given serious consideration. In this situation, as few as one satellite might be targeted, merely for propaganda purposes or to establish capability.

Analytic techniques are needed to perform trades to determine which self-protection approaches will provide positive gains in overall system effectiveness and cost. The first test that a given approach must pass in order to be given any further consideration is that it reduce the overall constellation life cycle cost, relative to the cost of the constellation with no survivability options. This means that the addition of the survivability subsystem(s) must allow, through enhanced survivability, sufficient reduction in the number of satellites to more than offset the cost increase per satellite due to increased complexity and weight. Once an approach has passed this initial hurdle, methods must be employed to optimize its performance vs key criteria, such as weight, and its best performance compared to that of alternate approaches. Specific analytic models are needed to perform the tasks of 1) optimizing satellite constellation (and related survivability equipment) sizing and 2) single-satellite survivability tactics optimization.

A variety of complicating factors make the full and complete formulation of such models difficult. These factors include launch costs (system weight/orbit), primary mission, survivability mission noninterference with primary mission, survivability equipment power and integration, survivability approach limitations (e.g., decoy patterns achievable), and the nature of the threat (interceptor vs directed beam, basing mode, engagement timeline, tracking/homing sensors). The number of factors and complexity of the relationships between them make it impractical to develop a high-fidelity model that applies generally to all situations.

The purpose of this paper is to present some first-order models that can be employed to solve some versions of problems 1) and 2) for a satellite for which the use of expendable decoys is the approach to survivability. In addition, some potentially fruitful directions for further model development are defined. The models presented here are best thought of as representative examples of how such conceptualization proceeds. Since the assumptions associated with these models are not highly system-specific, other potential application areas to ground, sea, or air engagements should be kept in mind while reviewing them.

Basic Modeling Assumptions

The following simplifying assumptions shall apply for all the models formulated next. The term *attacker* will be employed to describe a generic weapon, such as a laser or interceptor, used to attack a satellite. The term *engagement* will be used to refer to a single attack or multiple attacks, depending on the context.

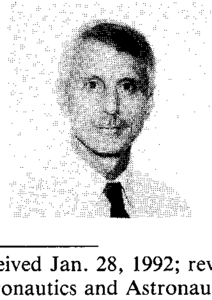
1) The only survivability tactic employed will be deployment of expendable decoys (along with associated maneuvers needed to establish spacing/geometry).

2) Each satellite in a given constellation will have the same number of decoys onboard at orbit insertion (i.e., identical satellites, same onboard equipment, same primary mission, etc.).

3) If a single attacker is targeted for a satellite during an engagement and the satellite releases α decoys, then a) the interceptor will kill one and only one of the $\alpha + 1$ available targets, and b) each of the $\alpha + 1$ available targets is equally likely to be killed. [*Note*: If one of the deployed bodies remains in the near vicinity of the satellite, to be employed to modify the signature(s) of the satellite, then the $\alpha + 1$ in a) and b) becomes α.]

4) There is no learning curve per engagement, i.e., 3b) is time-invariant.

The foregoing assumptions are, in fact, reasonable in various situations. For a threat involving an IR sensor, lightweight decoys can often be employed to provide good simulation of the satellite IR signature. For a threat involving radar, a



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lightweight expendable rf target (called a modifier) that has a much larger signature than the satellite can be deployed in the same resolution cell as the satellite in order to swamp its rf return. Decoys deployed in this situation would be selected to statistically match the combined return of the satellite and its modifier. The fine points of signature matching will not be addressed here.

Total Assault Models

A situation may arise, in an all-out war, for example, where all attackers will be used simultaneously to inflict as much damage as possible as quickly as possible on the portion of a satellite constellation within view from a fixed geographic location. The natural response to such a total assault is an all-out defensive effort, i.e., the deployment of all decoys by all satellites in the attack area. Since it is impossible to know a priori which portion of the constellation will be in the attack area when the attack is initiated, all satellites must be identically equipped with decoys. The modeling problem for this type of attack can be formulated as follows, using the total mass of the constellation as an optimization criterion.

Given:

1) N_a = number of attackers
2) M_s = mass of one satellite (without decoys)
3) M_d = mass of one decoy
4) All attackers attack all satellites in the attack area simultaneously and are evenly distributed over all targets (satellites and decoys).
5) Each satellite has the same number of decoys, and all satellites in the attack area release all of their decoys at the time of the attack.
6) Each attacker is restricted to attacking a given initially targeted satellite and/or the decoys it releases.
7) Two different attackers will not kill the same target (assume the total number of attackers does not exceed the total number of targets, to allow for some possible survivors).

Find:

The values of
p = initial number of satellites in the attack area
q = (initial) number of decoys per satellite
which minimize the attack-area constellation mass

$$M_t = pM_s + pqM_d \tag{1}$$

while satisfying the survivability requirement

$$P[X \geq L_0] \geq p_0 \tag{2}$$

where $P(E)$ denotes the probability of the occurrence of event E, X is the number of satellites in the attack area surviving the attack, p_0 is a predefined probability threshold, and L_0 is the number of satellites in the attack area required to have at least probability p_0 of surviving. Once this problem is solved, the total constellation mass can be obtained by multiplying M_t by the ratio of the number of satellites in the constellation to the number of satellites in the attack area (based on orbital altitude). This problem is solved as follows.

Probability Distribution Formulation

For a given choice of p and q, the evaluation of $P[X \geq L_0]$ can be performed as follows. If $N_a \geq p + pq$, then all attack-area satellites and decoys will be killed, so assume that

$$N_a < p + pq \tag{3}$$

The correct model for X depends on whether or not p exceeds N_a.

Model 1: $N_a \leq p$

For $N_a \leq p$, at most N_a satellites can be destroyed and

$$k_0 = p - N_a \tag{4}$$

are guaranteed to survive. The outcome of the attack on the ith attack-area satellite can be represented by a random variable Y_i defined by

$$Y_i = \begin{cases} 0 & \text{if the } i\text{th satellite is killed} \\ 1 & \text{otherwise} \end{cases} \tag{5}$$

Since $q + 1$ objects are equally likely to be killed,

$$P[Y_i = 0] = \frac{1}{q+1}, \qquad P[Y_i = 1] = \frac{q}{q+1} \tag{6}$$

for $i = 1, \ldots, N$. In this case, X is

$$X = k_0 + X' \tag{7}$$

where

$$X' = \sum_{1}^{N_a} Y_i \tag{8}$$

X' is binomially distributed with parameters $n = N_a$ and $\beta = q/(q+1)$, i.e.,

$$P[X' = i] = \begin{cases} \binom{n}{i}\beta^i(1-\beta)^{n-i} & i = 0, \ldots, n \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

The probability in Eq. (2) can be re-expressed as

$$P[X \geq L_0] = P[X' \geq L_0 - k_0] \tag{10}$$

For any integer k,

$$P[X' \geq k] = \sum_{i=k}^{N_a} P[X' = i] \tag{11}$$

Equation (10) can now be evaluated using Eqs. (11) and (9).

Model 2: $N_a > p$

For $N_a > p$, it is possible for all attack-area satellites to be destroyed, and the statistics are more complex. Elementary number theory (see Ref. 1) states that there are unique nonnegative integers M^* and r such that $r < p$ and

$$N_a = M^*p + r \tag{12}$$

It follows that r of the attack-area satellites will each have $M^* + 1$ attackers assigned to them, and the remaining $p - r$ of

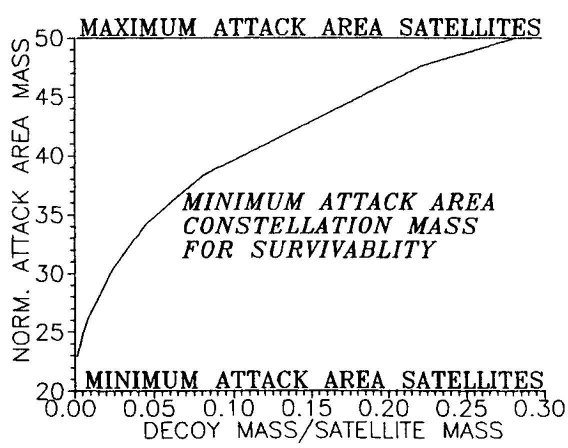

Fig. 1 Total assault minimum attack-area constellation mass: widely spaced targets.

the satellites will have M^* attackers assigned to them. Define the notation

$$M_1 = p - r, \qquad M_2 = r, \qquad M = M_1 + M_2 \tag{13}$$

and let $Y_1, \ldots, Y_M$ be defined by Eq. (5). For $i = 1, \ldots, M_1$,

$$P[Y_i = 1] = \begin{cases} \dfrac{q + 1 - M^*}{q + 1} & \text{if } M^* \le q + 1 \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

and for $i = M_1 + 1, \ldots, M$,

$$P[Y_i = 1] = \begin{cases} \dfrac{q - M^*}{q + 1} & \text{if } M^* \le q \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

In this situation,

$$X = X_1 + X_2 \tag{16}$$

where

$$X_1 = \sum_{i=1}^{M_1} Y_i, \qquad X_2 = \sum_{i=M_1+1}^{M} Y_i \tag{17}$$

X_1 is binomially distributed with parameters $n = M_1$ and $\beta_1 = (q + 1 - M^*)/\,(q + 1)$, and X_2 is binomially distributed with parameters $n = M_2$ and $\beta_2 = (q - M^*)/(q + 1)$. Notice that, as sums of independent random variables, X_1 and X_2 are independent. To evaluate Eq. (2),

$$P[X \ge k] = P[X_1 + X_2 \ge k] = \sum_{i=k}^{M} P[X_1 + X_2 = i]$$

Thus,

$$P[X \ge k] = \sum_{i=k}^{M} \sum_{j=0}^{i} P[X_1 = j]\, P[X_2 = i - j] \tag{18}$$

The probabilities on the right are given by

$$P[X_i = j] = \begin{cases} \dbinom{n}{i} \beta^i (1 - \beta)^{n-i} & i = 0, \ldots, n \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

with $n = M_i$ and $\beta = \beta_i$ for $i = 1, 2$.

Mass Minimization Reformulation

The attack-area constellation mass M_t can be normalized as follows. Let

$$\rho_d = M_d / M_s \tag{20}$$

$$\rho_t = p + pq\rho_d \tag{21}$$

Note that since $M_t = M_s \rho_t$, minimizing M_t is equivalent to minimizing ρ_t. The use of ρ_t amounts to assuming unity for the satellite mass; ρ_t can be thought of as the attack-area constellation mass measured in equivalent satellite count.

Algorithm Formulation

The evaluation of binomial coefficients should be done in the following manner, in order to avoid overflow problems (ℓn denotes the natural logarithm function and Γ the gamma function):

$$\binom{n}{i} = \exp\left[\ell n\,\Gamma(n + 1) - \ell n\,\Gamma(i + 1) - \ell n\,\Gamma(n - i + 1)\right]$$

The computation of the values of p and q that minimize ρ_t can be accomplished with recursion as follows. The smallest practical value to consider for p is $L_0 + 1$, and the maximum value for p is $L_0 + N_a$, since this many attack-area satellites will guarantee L_0 survivors. A reference set of optimal value pairs (p_i, q_i), $i = L_0 + 1, \ldots, L_0 + N_a$, is created efficiently as follows. For $i = L_0 + 1$, start with $q = 0$ and increase q by 1 until Eq. (2) is satisfied. This smallest value of q satisfying Eq. (2) is q_i for this first i and is optimal in the sense that any other choice of q for this p_i that satisfies Eq. (2) will produce a larger value of ρ_t. For the next value of i and recursively for all subsequent values of i, it is most efficient to start with q_{i-1} and decrement q until the first time that Eq. (2) is not satisfied. Then $q_i = q + 1$ is the optimal value sought. This process generates the pairs (p_i, q_i), $i = L_0 + 1, \ldots, L_0 + N_a$. Then, for any given value of ρ_d, Eq. (21) can be evaluated for all the pairs and the choice of p and q that minimizes ρ_t obtained.

An example of the outcome of this process is provided in Figs. 1–3, where the values $L_0 = 20$, $N_a = 30$, and $p_0 = 0.9$ were used. As one would expect, these figures show that the constellation mass saving is great only when the single decoy mass is a small fraction of a satellite mass. For this simple model, it is clear that the use of decoys at all becomes unattractive when the decoy mass approaches one-third of the satellite mass, a value that will make any satellite designer cringe. The optimal number of decoys varies inversely with the mass of a single decoy.

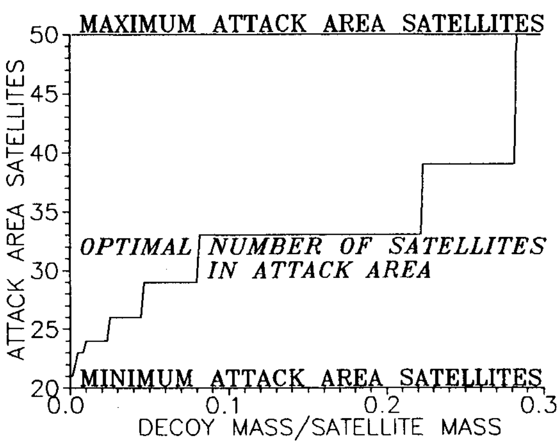

Fig. 2 Total assault optimal attack-area satellite count: widely spaced targets.

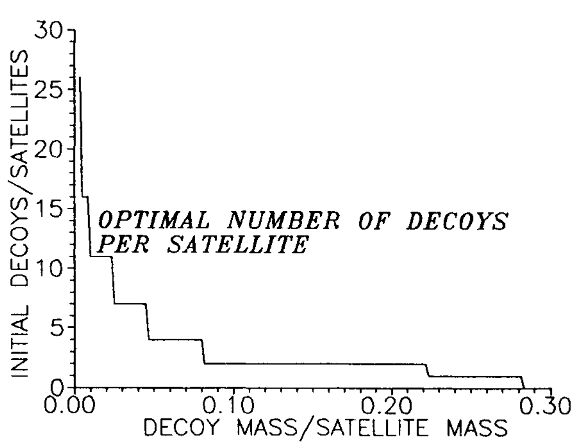

Fig. 3 Total assault optimal decoy count: widely spaced targets.

Alternate Model

It may be appropriate in a situation involving a large, closely spaced constellation to relax the assumption that each attacker may attack only the attack-area satellite (and its decoys) for which it is initially targeted. This results in X having a hypergeometric distribution (see Ref. 2) with density f given by

$$f(x) = \begin{cases} \binom{r_1}{x}\binom{r-r_1}{n-x}\binom{r}{n}^{-1} & \text{for } x = X_{\min}, \ldots, X_{\max} \\ 0 & \text{otherwise} \end{cases} \quad (22)$$

where $r_1 = p$, $r_2 = pq$, $r = r_1 + r_2$, $n = r - N_a$, and X takes on values only in the range from $X_{\min} = \max(0, n - r_2)$ to $X_{\max} = \min(n, r_1)$. The algorithm just defined still applies; an example for this model is shown in Figs. 4-6, with $L_0 = 400$, $N_a = 800$, and $p_0 = 0.9$. The same general trends apply. It is interesting to note that for this example the mass fraction breakpoint for usefulness of decoys has increased sizably over the preceding smaller-scale example.

Attrition Models

A situation may arise, during a period of international tension and limited hostilities, in which attrition tactics are employed. Rather than an all-out attack, the threat resources are employed in limited attacks that are drawn out over a long period of time. This results in a gradual erosion of the constellation and its mission effectiveness. A fundamental issue in constellation sizing is the selection of a primary sizing criterion: all-out assault or attrition. If the all-out assault criterion is used, then the best response with available resources to an attrition attack must be determined after the fact.

Local Attrition Models

If we assume that the number of satellites and decoys per satellite have been predetermined, what is the best way for a single satellite to use its decoys in an attrition situation? One decoy could be used per engagement, for example, or all decoys could be expended on the first engagement. One simple optimization criterion is to maximize the mean number of engagements required to kill the satellite. For those accustomed to trying to guarantee some predetermined probability of survival, the use of the mean in this manner may be unpalatable. It is important to recognize, however, that in the postulated situation the usual degrees of freedom are reduced and the attack scenario is qualitatively different. Instead of having a predefined fixed number of possible attackers and flexibility in constellation/decoys sizing, the long timeline can allow for the continued manufacturing or acquisition of attackers, and the constellation configuration is fixed. The best goal to set is to make the adversary pay the highest price possible in terms of resources expended. Maximizing the mean is a reasonable criterion to employ to accomplish this. In fact, no other reasonable criterion immediately suggests itself. For design and functional standardization, it is reasonable to assume that the same number of decoys will be released per engagement, with a possibly smaller number on the last deployment. Let

q = number of decoys on the satellite initially

α = number of decoys expended "per engagement" (to be optimized).

There exist nonnegative integers m and r such that $r < \alpha$ and

$$q = \alpha m + r \quad (23)$$

There are thus potentially $m + 1$ decoy releases, the first m releases deploying α decoys each, and the last deploying r decoys. If the satellite survives to the ith engagement, then that event may be represented by a random variable Y_i. For $i \leq m$, Y_i has equal probability of attaining any value in the set $\{1, \ldots, \alpha + 1\}$ where the outcome 1 corresponds to eliminating the target. Y_{m+1} takes values in the smaller set $\{1, \ldots, r + 1\}$, with sure kill on engagement $m + 1$ if $r = 0$.

The random variable of interest is X, the number of engagements required to eliminate the satellite. The probability density of X is

$$P[X = 1] = \frac{1}{\alpha + 1} \quad (24)$$

$$P[X = 2] = P[Y_2 = 1, Y_1 \neq 1] = \frac{1}{\alpha + 1}\frac{\alpha}{\alpha + 1} \quad (25)$$

$$P[X = 3] = P[Y_3 = 1, Y_2 \neq 1, Y_1 \neq 1] = \frac{1}{\alpha + 1}\left(\frac{\alpha}{\alpha + 1}\right)^2 \quad (26)$$

$$\vdots$$

$$P[X = m] = P[Y_m = 1, Y_i \neq 1, i = 1, \ldots, m - 1] = \frac{1}{\alpha + 1}\left(\frac{\alpha}{\alpha + 1}\right)^{m-1} \quad (27)$$

$$P[X = m + 1] = P[Y_{m+1} = 1, Y_i \neq 1, i = 1, \ldots, m] = \frac{1}{r + 1}\left(\frac{\alpha}{\alpha + 1}\right)^m \quad (28)$$

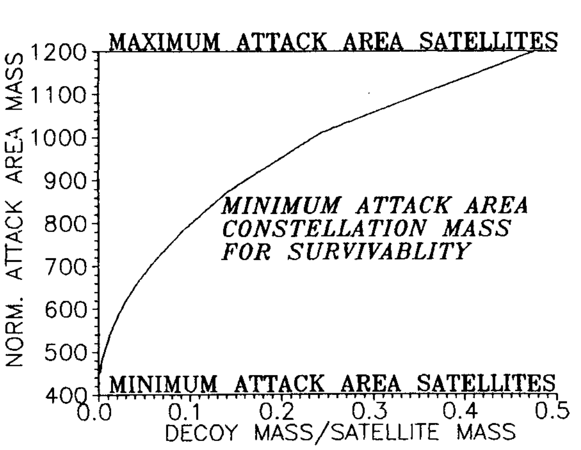

Fig. 4 Total assault minimum attack-area constellation mass: closely spaced targets.

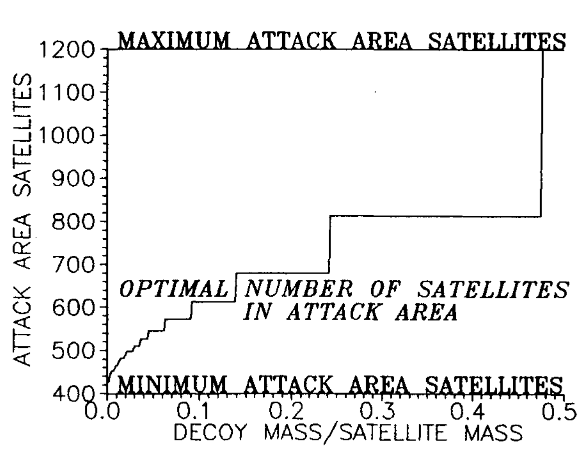

Fig. 5 Total assault optimal attack-area satellite count: closely spaced targets.

$$P[X = m + 2] = P[Y_i \neq 1,\ i = 1, \dots, m + 1]$$
$$= \frac{r}{r+1}\left(\frac{\alpha}{\alpha+1}\right)^m \tag{29}$$

Applying the definition of the mean and an arithmetic-geometric series formula to the aforementioned probabilities gives the mean $\overline{X}$ as

$$\overline{X} = 1 + \alpha + \left\{\left[m + \frac{r}{r+1} + \alpha(m-1)\right]\frac{\alpha}{\alpha+1} - \alpha m\right\} \times \left(\frac{\alpha}{\alpha+1}\right)^{m-1} \tag{30}$$

For a given value of q, the last equation can be evaluated for $\alpha = 1, \dots, q$ to determine the maximum value of $\overline{X}$. The results of this procedure are illustrated in Fig. 7. The top curve in Fig. 7 indicates the number of decoys to deploy per engagement in order to maximize the mean engagements required to kill the satellite, as a function of the total decoys available prior to any attack. The bottom curve is the actual maximum value of the mean number of engagements required to kill. The maximum value of $\overline{X}$ clearly increases as the initial number of decoys is increased, but the rate of increase is unfortunately fairly slow. The probability distribution for X defined here allows straightforward calculation of the dispersion of X about the mean.

The foregoing situation can be altered by always employing one of the expended objects as a satellite signature modifier, thus creating one less targetable object per engagement. This results in a peculiarity: If at some point only one expendable is left, then the satellite will be killed on the next engagement. Equation (30) is replaced by

$$\overline{X} = \alpha + \left\{\left[\alpha(m-1) + 1\right]\frac{\alpha-1}{\alpha} - (\alpha-1)m\right\} \times \left(\frac{\alpha-1}{\alpha}\right)^{m-1} \tag{31}$$

for $r = 0$, and by

$$\overline{X} = \alpha + \left\{\left[\alpha(m-1) + 2 - \frac{1}{r}\right]\frac{\alpha-1}{\alpha} - (\alpha-1)m\right\} \times \left(\frac{\alpha-1}{\alpha}\right)^{m-1} \tag{32}$$

for $0 < r < \alpha$. The application for this situation of the same procedure for finding the value of α that maximizes $\overline{X}$ is shown in Fig. 8. As would be expected, the maximum value of the mean has decreased compared to the situation shown in Fig. 7.

Global Attrition Models

As a practical matter, the loss of more than some predefined percentage of the original constellation will result in an unacceptable level of mission performance. This shifts the focus from the individual satellite to the constellation. The following simplifying assumptions will be made:

5) The satellites are exposed to attack in a fixed predetermined sequence.

6) Only one attack will be directed at each satellite per iteration through the sequence (like ducks in a shooting gallery!).

These assumptions are quite reasonable for a ground-based threat. The use of one expendable per engagement as a satellite modifier will also be assumed, as in the last situation. Unfortunately, the sequencing of the satellites seems to make a Monte Carlo approach unavoidable for determining the value of α that maximizes the mean number of engagements required to eliminate the defined percentage of the original constellation. Equation (23) still applies here. If a satellite is not eliminated before the ith engagement against it, then its probability of being eliminated on the ith engagement can be represented by a random variable Y, with

$$Y = \begin{cases} 1 & \text{if the satellite survives} \\ 0 & \text{if the satellite is killed} \end{cases} \tag{33}$$

$$P[Y = 0] = \begin{cases} 1/\alpha & \text{if } i \leq m \\ 1/r & \text{if } i = m + 1 \text{ and } 1 < r \\ 1 & \text{otherwise} \end{cases} \tag{34}$$

In order to preserve space, the reader is referred to the definition of the Monte Carlo procedure in Ref. 3. The computation of dispersion about the mean can readily be added to the procedure in Ref. 3. The results of the procedure are illustrated in Fig. 9, where the predefined percentage is 33%, the number of satellites is 30, and q ranges from 1 to 10. The gain in effectiveness is fairly dramatic for a small number of decoys, but less so as the number of initial decoys grows larger. The optimal number of decoys per attack is slowly varying and without a definite trend.

Up to this point, the attrition models developed have assumed that the number of satellites p and expendables per satellite q have been predetermined by some criterion other

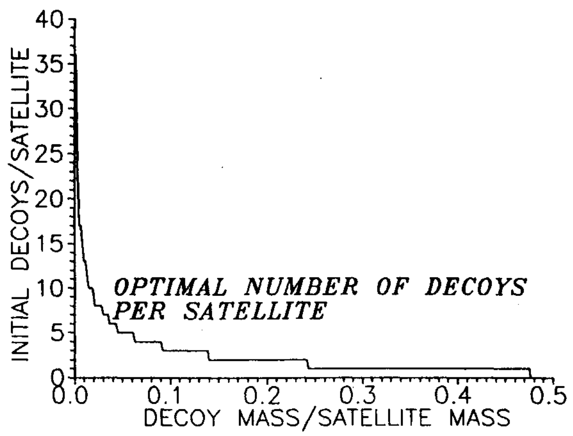

Fig. 6 Total assault optimal decoy count: closely spaced targets.

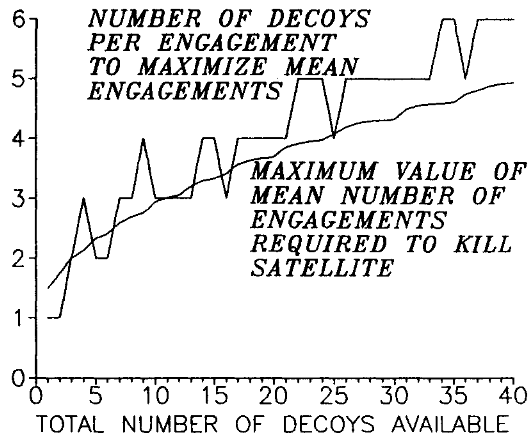

Fig. 7 Local attrition model maximized mean engagements: no satellite modifier.

than attrition. The last model leads in a natural way to a method for optimizing the choice of p and q relative to an attrition attack, rather than an all-out assault. Assumptions 1–6 will apply (excluding the use of satellite modifiers), as well as everything under the "Given" heading at the beginning of the "Total Assault Models" section (except the fourth and fifth assumptions). The problem then is to find the values of

p = number of satellites
q = number of decoys per satellite
α = number of decoys deployed per satellite per engagement

which minimize the constellation mass

$$M_t = pM_s + pqM_d \tag{35}$$

while satisfying the survivability requirement

$$P[X \geq L_0] \geq p_0 \tag{36}$$

where X is the number of satellites surviving the total extended attack by all N_a attackers, p_0 a predefined probability threshold, and L_0 the number of surviving satellites required with probability p_0.

The foregoing problem is solved as follows.

Probability Distribution Formulation

For a given choice of p, q, and α, the evaluation of $P[X \geq L_0]$ can be performed as follows. If $N_a \geq p + pq$, then all satellites will be killed, so assume that

$$N_a < p + pq \tag{37}$$

The correct model for X depends on whether or not p exceeds N_a.

Model 1: $N_a \leq p$

For $N_a \leq p$, at most N_a satellites can be destroyed and

$$k_0 = p - N_a \tag{38}$$

are guaranteed to survive. The sequencing of the satellites in this case is immaterial, since the initial pass through the sequence uses up all the attackers. The ith satellite attacked can be represented by a random variable Y_i defined by

$$Y_i = \begin{cases} 0 & \text{if the } i\text{th satellite is killed} \\ 1 & \text{otherwise} \end{cases} \tag{39}$$

In this situation, it is obvious that $\alpha = q$ is the best choice for α to maximize $P[X \geq L_0]$. Since $q + 1$ objects are equally likely to be killed, the method of evaluation of $P[X \geq L_0]$ is identical to that used in model 1 of the "Total Assault Models" section.

Model 2: $N_a > p$

For $N_a > p$, it is possible for all satellites to be destroyed, the attack exposure sequence begins to cycle, and the statistics are more complex. There are unique nonnegative integers m and r, $r < p$, and unique nonnegative integers m' and r', $r' < \alpha$, such that

$$N = mp + r \tag{40}$$

$$q = m'\alpha + r' \tag{41}$$

It follows that r of the satellites will each have up to $m + 1$ attackers attack them, and the remaining $p - r$ of the satellites will have up to m attackers attack them. It also follows that each satellite is capable of up to m' engagements with α decoys released, plus one additional degraded engagement with r' decoys released if $r' > 0$. Sure kill of all satellites may or may not occur, based on the values of m, m', r, and r'.

If a satellite is not eliminated before the ith engagement against it, then its probability of being eliminated on the ith engagement can be represented by a random variable Y, with

$$Y = \begin{cases} 0 & \text{if the satellite is killed} \\ 1 & \text{if the satellite survives} \end{cases} \tag{42}$$

$$P[Y=0] = \begin{cases} 1/(\alpha+1) & \text{if } i \leq m' \\ 1/(r'+1) & \text{if } i = m'+1 \text{ and } 0 < r' \\ 1 & \text{otherwise} \end{cases} \tag{43}$$

This ith engagement cannot possibly occur for $i = m + 1$, except for r out of the total number p of satellites. The evaluation of $P[X \geq L_0]$ requires the use of the Monte Carlo method.[4-6] An example of the outcome of this process is provided in Figs. 10–12, where the values $L_0 = 20$, $N_a = 30$, and $p_0 = 0.9$ were used. The total constellation mass and total number of decoys can be decreased quite a lot if very lightweight decoys can be employed.

Model Refinements Needed

The models developed here can be employed for first-order analyses. Although they may not apply in a specific situation, they provide "how-to" examples for developing the model appropriate to a given problem and perhaps an initial guess to reduce the solution space to be searched with a computationally more intensive detailed model.

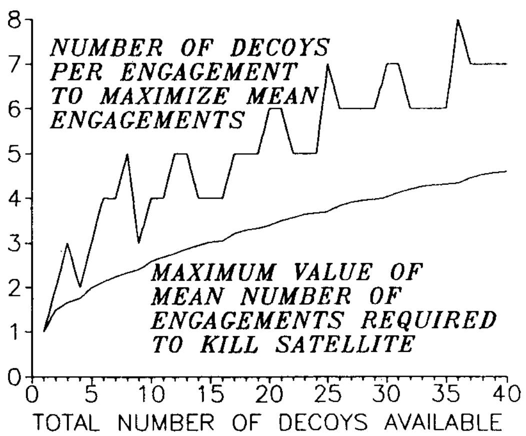

Fig. 8 Local attrition model maximized mean engagements: with satellite modifier.

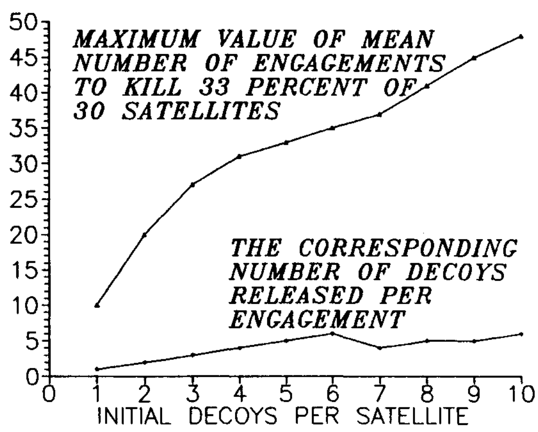

Fig. 9 Global attrition model maximized mean engagements: with satellite modifier.

Unequal Probabilities

The assumption of equal kill probabilities, assumption 3b, can readily be replaced by an assumption of the satellite being either more or less likely to be killed than a decoy. For example, ultralightweight decoys could be analyzed. A large number of such "traffic" decoys would not have to individually have as much chance of being killed as the satellite to be collectively effective. This model modification will allow a more meaningful trade between very lightweight and heavier decoys.

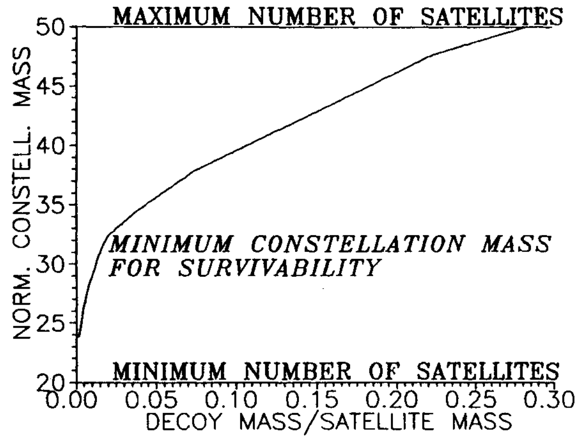

Fig. 10 Global attrition minimum constellation mass: no satellite modifier.

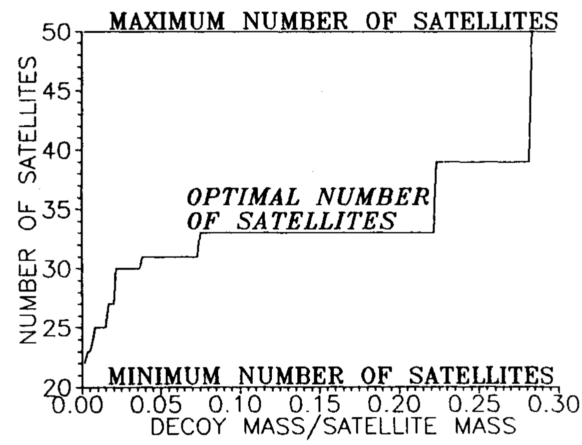

Fig. 11 Global attrition optimal satellite count: no satellite modifier.

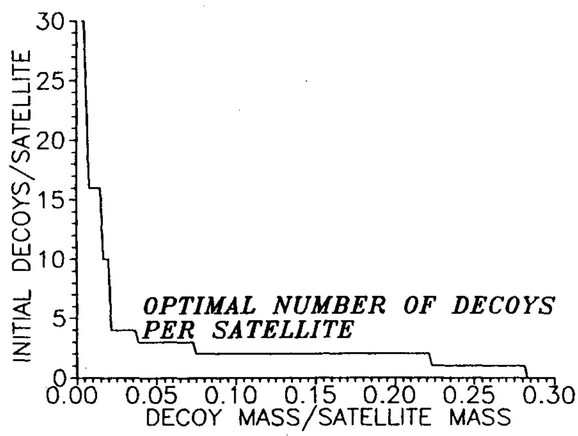

Fig. 12 Global attrition optimal decoy count: no satellite modifier.

Fuel Consumption Modeling

When an attack is directed against a specific satellite, it and its decoys must be deployed in a pattern suitable for making certain that no more than one target can be killed. In addition, at least the satellite ought to be removed from its nominal trajectory. Practical limitations on how rapidly a dispersal pattern can be created place upper limits on how many decoys can be deployed effectively per engagement. Due to the slow response time and fuel inefficiency associated with using the satellite to motor about in velocity space dropping decoys, separate ejection/dispersion systems would typically be employed to deploy the decoys. The weight of these systems must be considered part of the weight associated with the decoys, a factor easily handled by the models developed here. What is needed is to consider the total delta-velocity capacity needed by the satellite to support the total number of possible engagements. Since the fuel required to produce the delta-velocity for a given engagement is a function of the amount of fuel mass and decoy system mass remaining, the total fuel required for a given choice of total initial decoys and decoys per engagement must be solved for recursively. The models can be extended to contain this recursion at considerable computational expense. It should be possible to use a simple fuel consumption curve-fit nonrecursive model to get close initial estimates before the complete model is invoked. Note that, with a given expendable's residual mass, the satellite delta-velocity required per engagement is essentially independent of the number of decoys deployed. This fact will tend to bias optimal solutions toward using more decoys per engagement.

Spoofing

An operational concern for satellite survivability tactics is the concept of spoofing, in which an attack is somehow simulated in order to trigger the use of expendables for self-protection. For an interceptor attack, it is not clear how this would be accomplished without expending resources comparable to those consumed by an actual attack. Some important parameters for spoofing include the quality of data available to the defense and the response time available for decision making.

In an all-out assault, spoofing can be handled if an assessment can made a priori of both the threat attack and spoofing attack resources. Then the number of threat attackers plus the number of attack simulators can be used instead of the number of threat attackers only, in sizing the constellation and decoy count. More detailed knowledge in a given situation should allow improvement over this rather brute force approach.

In an attrition attack, the algorithms such as those developed here have an innate robustness for spoofing. The emphasis in attrition is inherently on preserving expendable resources, while maximizing the number of attacks, real or simulated, required to kill a satellite (or constellation fraction). Again, the knowledge of spoofing resources can be readily factored into sizing the constellation/expendables, or in defining attrition tactics.

In addition to erosion of resources and psychological impact, spoofing could be employed to gain insight into expendables performance and to develop discriminants. The use of a modifier to swamp the signature (particularly rf) of the satellite and the accompanying use of decoys to statistically match the combined satellite/modifier signature represent an approach that denies significant learning curve development, thus supporting the earlier modeling assumption.

Conclusion

Models have been developed that address satellite constellation/expendables optimal sizing and optimal expendables tactics. The focus has been on the use of expendable decoys. Simplifying assumptions have been made, such as the satellite and its decoys having equal likelihood of being killed in a given engagement, with the objective of laying the groundwork for more comprehensive model development and providing first-order analytic tools. Models were presented for optimal sizing for an all-out attack against a portion of the constellation situated over a fixed geographic location, for both large and small constellations. Two categories of attrition attack optimal sizing models were derived. The first dealt with adjusting tactics to an attrition attack for a constellation already sized based on another attack assumption, such as an all-out assault. The other focused on optimal sizing with an attrition attack as the primary assumption. All modeling results strongly indicate the value of lightweight decoys. Areas for further model refinements, such as fuel consumption modeling and greater flexibility in statistical assumptions, have been identified. Related information can be found in Ref. 7.

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